NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science; Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSOC; 07BAMS	**LEVEL:** 6
COURSE CODE: LIA601S	**COURSE NAME:** LINEAR ALGEBRA 2
SESSION: JANUARY 2018	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 90

SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
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MODERATOR:	Dr O. SHUUNGULA

INSTRUCTIONS
1. Examination conditions apply at all times. **NO** books, notes, or phones are allowed.
2. Answer **ALL** the questions and number your answers clearly and correctly.
3. Show clearly all the steps used in the calculations.
4. Write clearly and neatly.
5. All written work must be done in dark blue or black ink.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1. [25 MARKS]

1.1 Let $T : P_1 \to \mathbb{R}^2$ be a mapping defined by

$$T[p(x)] = [p(0), p(1)].$$

(a) Find $T[1 - 2x]$. [1]

(b) Show that T is a linear mapping. [5]

(c) Is T one-to-one? Explain your answer. [6]

1.2 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator for which $T(1, 2) = (3, -1)$ and $T(0, 1) = (2, 1)$. By noting that $\{(1, 2), (0, 1)\}$ is a basis of $\mathbb{R}^2$, find a formula for $T(x, y)$, and then use the formula to compute $T(3,\ 5)$. [7]

1.3 Let F and G be the linear operators on $\mathbb{R}^2$ defined by

$$F(x, y) = (x + y, 0) \quad \text{and} \quad G(x, y) = (-y, x).$$

Find formulas defining the following linear operators:

(a) $3F - 2G$. [2]

(b) $F \circ G$. [2]

(c) F^2. [2]

QUESTION 2. [23 MARKS]

2.1 Consider the linear operator G on $\mathbb{R}^2$, defined by $G(x, y) = (3x + 4y, 2x - 5y)$, and the basis $S = \{(1, 2), (2, 3)\}$ in $\mathbb{R}^2$. Find the matrix representation of G relative to S. [8]

2.2 Consider the bases

$$S_1 = \{p_1, p_2\} = \{6 + 3x,\ 10 + 2x\} \quad \text{and} \quad S_2 = \{q_1, q_2\} = \{2,\ 3 + 2x\}$$

for P_1, the vector space of polynomials of degree ≤ 1.

(a) Find the transition matrix P from S_1 to S_2. [8]

(b) Compute the coordinate vector $[p]_{S_1}$, where $p = -4 + x$, and use the transition matrix you obtained in part (a) above to compute $[p]_{S_2}$. [7]

QUESTION 3. [22 MARKS]

3.1 Suppose that the characteristic polynomial of some square matrix A is found to be

$$p(\lambda) = (\lambda - 1)(\lambda - 3)^2(\lambda - 4)^3.$$

(a) What is the size of the matrix A? [2]

(b) Is the matrix A invertible? [2]

(c) How many eigenspaces does A have? [2]

Explain your answers.

3.2 Suppose $A = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

(a) Confirm that P diagonalises A, by finding P^{-1} and directly computing $P^{-1}AP = D$. [9]

(b) Hence, find A^{1000}. [7]

QUESTION 4. [20 MARKS]

4.1 Let $\mathbf{x}^T A\mathbf{x}$ be a quadratic form in the variables $x_1, x_2, \cdots, x_n$, and define $T : \mathbb{R}^n \to \mathbb{R}$ by $T(\mathbf{x}) = \mathbf{x}^T A\mathbf{x}$. Show that $T(\mathbf{x} + \mathbf{y}) = T(\mathbf{x}) + 2\mathbf{x}^T A\mathbf{y} + T(\mathbf{y})$ and $T(c\mathbf{x}) = c^2 T(\mathbf{x})$, for any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ and $c \in \mathbb{R}$. [6]

4.2 Consider the equation $5x_1^2 - 4x_1x_2 + 8x_2^2 = 36$.

(a) Re-write the equation in the matrix form $\mathbf{x}^T A\mathbf{x} = 36$, where A is a symmetric matrix. [4]

(b) Given that the matrix

$$P = \begin{bmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$$

orthogonally diagonalises A, use a suitable variable transformation to place the conic in standard position and, hence, identify the conic section represented by the equation. [10]

END OF QUESTION PAPER